GUIDED WAVE BEHAVIOR ANALYSIS IN MULTI-LAYERED INHOMOGENEOUS ANISOTROPIC PLATES

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INTRODUCTION

Guided waves behave much differently in inhomogeneous anisotropic plates than in homogeneous anisotropic plates. It has been reported before that guided waves in inhomogeneous plates tend to follow preferred directions based on the location of ply-groups as well as the orientation of the fibers [1]. The pattern obtained by imaging the leaked energy into the surrounding fluid (earlier called as Plate Wave Flow Patterns) has been shown to indicate fiber orientations [2]. In this paper, a model based on the Thomson-Haskell transfer matrix is employed to obtain the internal distributions of the energy vector within the inhomogeneous plate. Based on the theoretical results, the plate wave flow patterns can be predicted and compared with the experimental results. The results provide insight into the understanding of the generation mechanism of guided wave mode patterns in inhomogeneous plates.

THEORETICAL BACKGROUND

Consider a general anisotropic multilayered plate immersed in water. The plate consists of an arbitrary number of general anisotropic layers n, rigidly bonded at their interfaces as shown in Figure 1. Without losing generality, it is assumed that leaky plate waves propagate along the x_1 direction. First consider the plane wave propagation in a general anisotropic multi-layered structure as shown in Figure 1. The displacements of the wave are given by

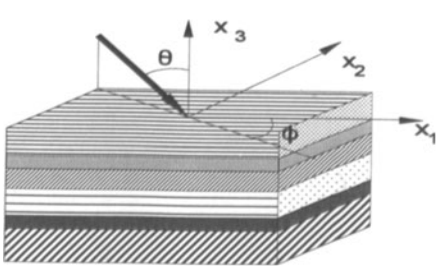

Figure 1. Representation of the theoretical multilayered model.

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Equation 1.

$$
\begin{aligned}
&u_i = A U_i e^{j(k_i x_i - \omega t)} \\
\textit{where}\quad & i = 1,2,3 \\
& A = \textit{amplitude} \\
& U_i = \textit{displacement vector} \\
& k_i = \textit{cosine components of wave number} \\
& \omega = \textit{circular frequency}
\end{aligned}
\tag{1}
$$

Based on the Thomson-Haskell transfer matrix method [3], Equation 2 gives the relationship between the displacements and stresses at the top interface and the bottom interface of the plate.

$$X^t = T X^b \tag{2}$$

where

$$X^t = \begin{bmatrix} u_1^t & u_2^t & u_3^t & \sigma_{33}^t & \sigma_{23}^t & \sigma_{13}^t \end{bmatrix}^T$$

$$X^b = \begin{bmatrix} u_1^b & u_2^b & u_3^b & \sigma_{33}^b & \sigma_{23}^b & \sigma_{13}^b \end{bmatrix}^T$$

In order to calculate the reflection and transmission coefficients for a plane wave incident on a multilayered anisotropic plate, an anisotropic plate between two different fluid half spaces must first be considered. In this situation, the shear stresses at upper and lower interfaces are zero. Thus, the boundary conditions at the top and the bottom interfaces are given by Equation 3.

$$
\begin{aligned}
&u_3^{ft} = u_3^t; \qquad \sigma_{33}^{ft} = \sigma_{33}^t; \qquad \sigma_{13}^t = \sigma_{23}^t = 0; \\
&u_3^{fb} = u_3^b; \qquad \sigma_{33}^{fb} = \sigma_{33}^b; \qquad \sigma_{13}^b = \sigma_{23}^b = 0;
\end{aligned}
\tag{3}
$$

By modifying Equation 2 and considering the boundary conditions (Equation 3), the following equation is obtained [4].

$$B^I = M A \tag{4}$$

where

$$B^I = \begin{bmatrix} 0 & 0 & 0 & 0 & u_1^I & \sigma_{33}^I \end{bmatrix}^T$$

$$M = \begin{bmatrix} -1 & 0 & t_{11} & t_{12} & 0 & s_1 \\ 0 & -1 & t_{21} & t_{22} & 0 & s_2 \\ 0 & 0 & t_{51} & t_{52} & 0 & s_5 \\ 0 & 0 & t_{61} & t_{62} & 0 & s_6 \\ 0 & 0 & t_{31} & t_{32} & -a_1 & s_3 \\ 0 & 0 & t_{41} & t_{42} & -a_2 & s_4 \end{bmatrix}$$

$$A = \begin{bmatrix} u_1^t & u_2^t & u_1^b & u_2^b & A_1^r & A_1^t \end{bmatrix}^T$$

Plate waves propagate in plate without external excitation, and this condition is satisfied when the fluid load B' equals to zero. The determinant of M must be zero for a non-zero solution of A.

$$\det(M) = 0 \tag{5}$$

In general for a given frequency and plate thickness, the roots of the above equation k_l are complex. The inverse of the real part of the root $Re(k_l)$ is the phase velocity of the leaky plate wave while the imaginary part of the root $Im(k_l)$ is the attenuation of the plate wave. It should be noticed that the attenuation of leaky plate waves consists of two parts: one is due to the viscosity which absorbs the energy of the plate wave , and the other represents the amount of energy which leaks to the surrounding fluid.

THE GUIDED WAVE POWER FLOW VECTOR

The particle velocities can easily be obtained by solving Equation 4 and taking the derivative of particle displacements with respect to time t.

$$v_i^{\alpha} = -j\omega u_i^{\alpha}$$
$$\textit{where} \quad \alpha = a \ \textit{wave mode} \tag{6}$$
$$i = 12,3 \ \textit{corresponding to} \ x_1, \ x_2, \ x_3.$$

The stress field can be obtained from Hooke's law given by Equation 7.

$$\sigma_{ij}^{\alpha} = C_{ijkl}\epsilon_{kl}^{\alpha} \tag{7}$$

Thus, the power flow vectors are defined as given by Equation 8 [5].

$$p_i^{\alpha} = \frac{1}{2}\sigma_{ij}^{\alpha}v_j^{\alpha *} \tag{8}$$

For each lay-up, the power flow magnitude and direction of each partial wave mode is determined in each layer and tabulated as shown in Table 1.

BEAM MODEL SIMULATION

In order to compare theoretical results with experimental image results, a simple beam image simulation model was developed. The energy value calculated from the plane wave model is assumed to have a guassian distribution across the power flow direction. Thus, the energy value at each grid point was computed using Equation 9

$$e_g(x,y) = \sum_{j=1}^{m}\sum_{i=1}^{n} \frac{E_{ij}}{r\sqrt{2\pi\sigma}} e^{-\frac{x}{2\sigma}} \tag{9}$$

Table1. Numerical results of guided waves propagation in $(45_5/-45_5)_s$ laminate.

	Propagation Angle			Power Flow Magnitude (E_{ij})GW/m^2		
	Layer 1	Layer 2	Layer 3	Layer 1	Layer 2	Layer 3
Long. (L)	40	-40	40	0.07	0.06	0.02
Shear (Tf)	32	-32	32	0.06	0.05	0.02
Shear (Tl)	17	-17	17	0.09	0.04	0.02

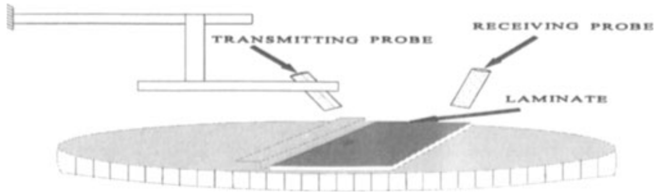

Figure 2. Experimental set-up.

In Equation 9, E_{ij} is the energy value of a partial wave mode at each layer and is obtained from the magnitude of the power flow vector as shown in Table 1; n is the number of the layers; m is the number indicating the partial wave mode; r is the projection of the distance between the grid point and the source on the power flow direction; x is the distance of the grid point from the beam center; α is the angle of divergence, and σ is the variance which is a function of r and α as shown by Equation 10.

$$\sigma=\sigma_{init}+r\cos(\alpha) \tag{10}$$

The theoretical image is obtained by plotting the $e_g(x,y)$ function and using a grey scale to represent the magnitude of the sum of the energy from individual partial wave modes.

NUMERICAL RESULTS AND DISCUSSION

The experimental study of plate wave propagation in inhomogeneous composite laminates has been conducted by Sullivan et. al. [1, 2]. The experimental set-up is shown in Figure 2 , where the transmitting probe was fixed while the receiving probe scanned the laminate. The leaky plate waves were generated with a transmitter incidence at 10°. The experiments indicated that the plate waves followed the preferred fiber direction [2]. There were three specimens used. All the specimens have thickness of 2.72 mm. The layup patterns of the specimens are : A = $(45_5/\text{-}45_5)_s$, B = $(90_2/45_4/135_4)_s$, and C = $(60_5/150_5)_s$.

Four cases were studied for the comparison between theoretical results and experimental results. Figure 3a & 3b show both theoretical and experimental results of guided wave propagation in Laminate A using a frequency of 1 MHz. Theoretical and experimental results show similar patterns. It should be noted that besides the two strong power flow propagation patterns along the ± 40°, there are also weaker beam propagations along smaller angles. The theoretical results indicate that the stronger beams are combined by quasi-longitudinal waves (± 40°) and fast quasi-shear waves (± 32°), while the weaker beams are slow quasi-shear waves. Figure 3d shows the theoretical result of the guided waves propagation in Laminate A using a frequency of 0.5 MHz. From the image one can see that most of the energy propagates along the 45° layers. This agrees with the experimental results shown in Figure 3c. Figure 3e & 3f show

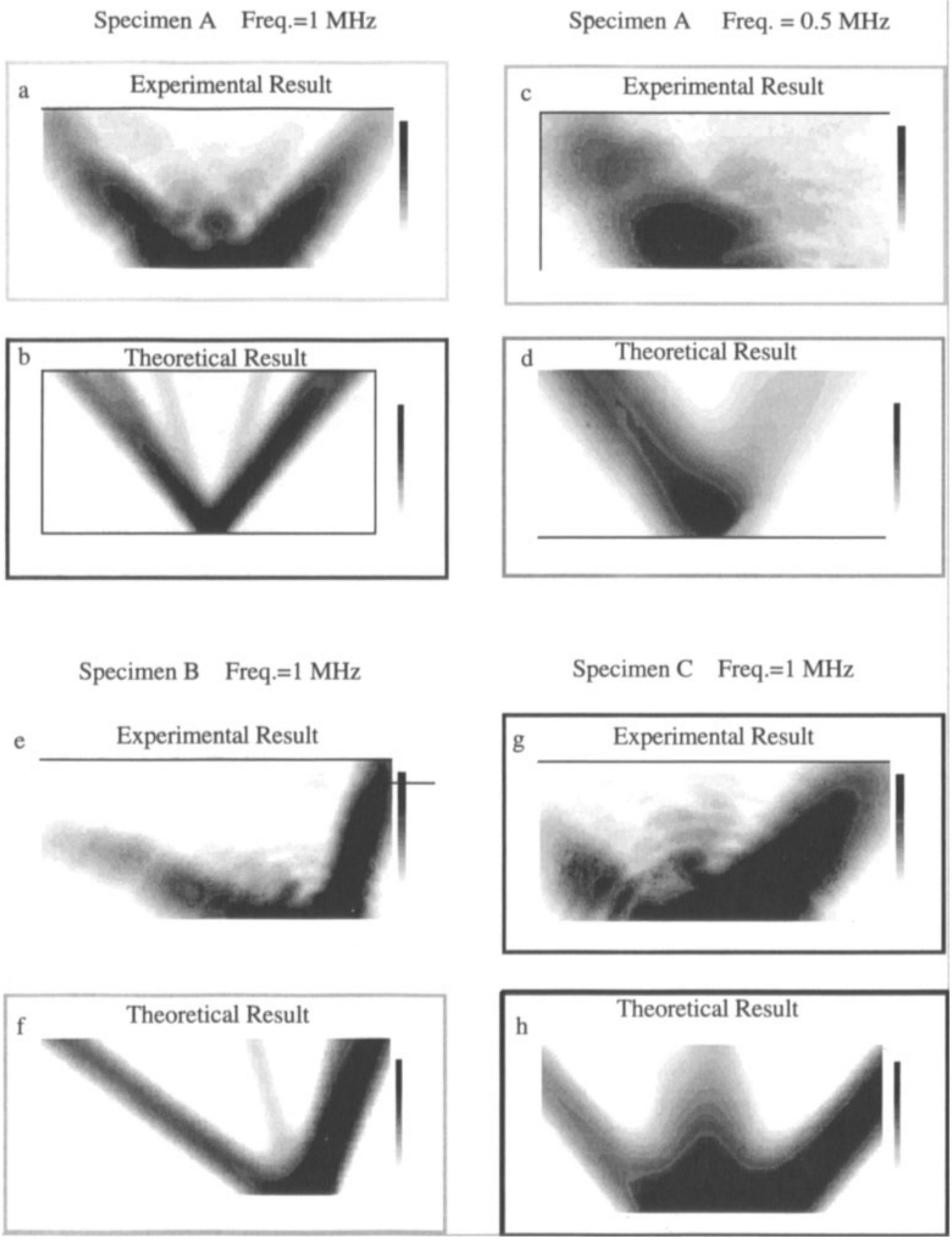

Figure 3. The comparison of experimental results with theoretical results.

the guided wave propagation in Laminate B, and Figure 3g & 3h show the results of guided waves in Laminate C. In both cases the theoretical results are similar to the experimental results.

CONCLUSIONS

It is clear from the above analysis that ultrasonic guided wave generation and propagation are different in inhomogeneous anisotropic plate than in homogeneous anisotropic plates. Ultrasonic guided wave behavior in inhomogeneous composite plates indicate that the beam splits. This can be predicted by using superposition of the partial wave power flow vector in individual layers.

The correlation between the experimental results and the theoretical prediction show that the generation of guided wave modes in inhomogeneous media are related to power flow distribution of individual partial wave modes.

ACKNOWLEDGMENT

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